Section 5.3

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(10b) Sketch a graph of $y = f(x)$.

(a) $f(x) = 4^x$ (b) $f(x) = \left(\frac{1}{4}\right)^x$

Part-a

Given $f(x) = 4^x$

$f(-1) = 4^{-1} = 1/4 = 0.25$

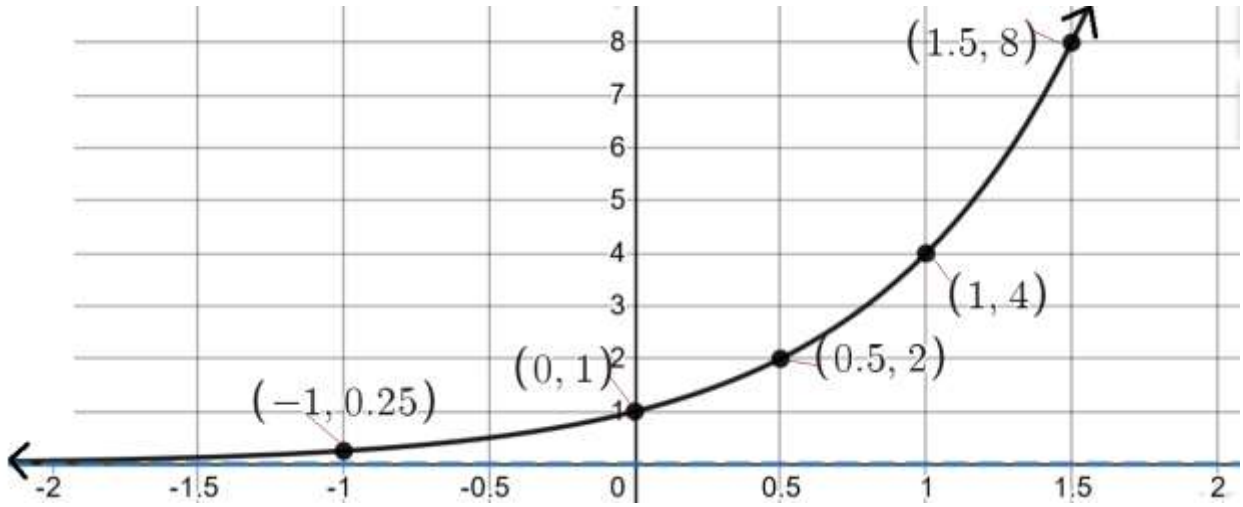
$f(0) = 4^0 = 1$

$f(1/2) = 4^{1/2} = \sqrt{4} = 2$

$f(1) = 4^1 = 4$

$f(3/2) = 4^{3/2} = \sqrt{4^3} = \sqrt{64} = 8$

Based on the calculations above, the graph contains the points (-1, 0.25), (0, 1), (0.5, 2), (1, 4), (1.5, 8). We also include the horizontal asymptote y= 0 to guide us in graphing the function



Part-b

Given $f(x) = \left(\frac{1}{4}\right)^x$

$f(-3/2) = \left(\frac{1}{4}\right)^{-3/2} = 4^{3/2} = \sqrt{4^3} = \sqrt{64} = 8$

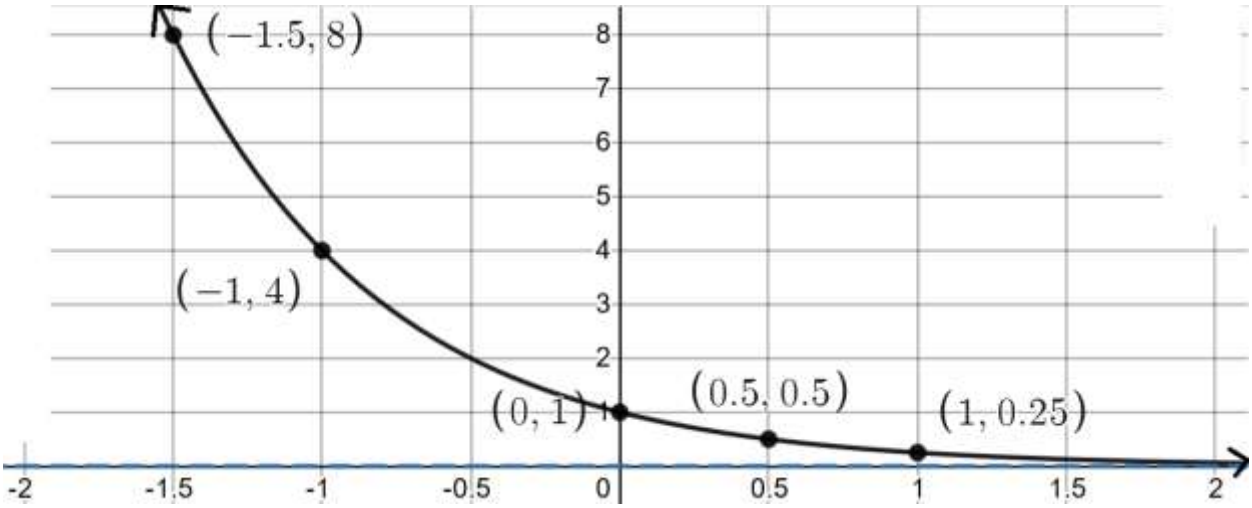
$f(-1) = \left(\frac{1}{4}\right)^{-1} = 4$

$f(0) = \left(\frac{1}{4}\right)^{0} = 1$

$f(1/2) = \left(\frac{1}{4}\right)^{1/2} = \sqrt{1/4} = 1/2 = 0.5$

$f(1) = \left(\frac{1}{4}\right)^{1} = 1/4 = 0.25$

Based on the calculations above, the graph contains the points (-1.5, 8), (-1,4), (0,1), (0.5,0.5), (1, 0.25). We also include the horizontal asymptote y= 0 to guide us in graphing the function



107

(107) Find C and a so that $f(x) = Ca^x$ satisfies the given conditions.

(a) $f(0) = \frac{3}{2}$, $f(-1) = 12$ (b) $f(1) = 18$, $f(-1) = 2$

Part-a

$f(0) = Ca^0 = C = 3/2$

$f(-1) = \frac{3}{2}(a)^{-1} = 12$

Multiply both sides by 2/3

$(a)^{-1} = 8$

$\frac{1}{a} = 8$

Get the reciprocal of both sides

$a = 1/8$

Answer: $C = \frac{3}{2}$ and $a = \frac{1}{8}$

Part-b

$f(1) = Ca^1 = Ca = 18$

$f(-1) = C(a)^{-1} = 2$

Multiply the 2 equations above

$C^2 = 36$

Take the positive square root of both sides

$C = 6$

$6a = 18$

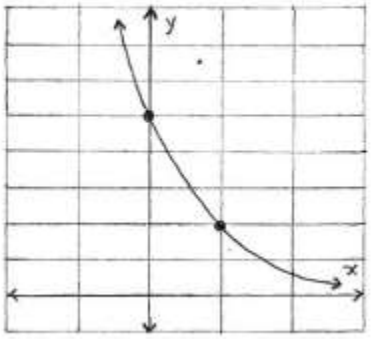
Divide both sides by 6

Answer: $C = 6$ and $a = 3$

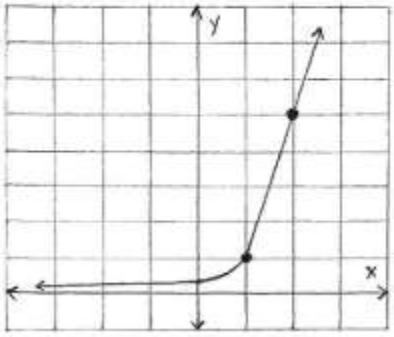
108

(108) Use the graph of $y = Ca^x$ to determine values for C and a.

(a)



(b)



Part-a

$f(0) = Ca^0 = C = 5$
$f(1) = 5(a)^1 = 2$
Divide both sides by 5
$a = \frac{2}{5}$
Answer: $C = 5$ and $a = \frac{2}{5}$

Part-b

$f(1) = Ca^1 = Ca = 1$
$f(2) = C(a)^2 = 5$
Divide the second equation by the first equation
$a = 5$
$5C = 1$
$C = 1/5$
Answer: $C = \frac{1}{5}$ and $a = 5$

109

(109) Use the compound interest formula to approximate the future value of each amount.

(a) \$1,500 at 1.2% compounded monthly for 10 years.
(b) \$20,000 at 5% compounded daily for 25 years.
(c) \$20,000 at 5% compounded continuously for 25 years.

Part-a

$$A_p(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

$$= 1500\left(1+\frac{0.012}{12}\right)^{12\times 10}$$

$$= 1500(1.001)^{120}$$

$$\approx \$1691.14$$

Part-b

$$A_p(t) = P\left(1+\frac{r}{n}\right)^{nt}$$

$$= 20000\left(1+\frac{0.05}{365}\right)^{365\times 25}$$

$$\approx \$69800.88$$

Part-c

$$A_c(t) = Pe^{rt}$$

$$= 20000e^{0.05\times 25}$$

$$= 20000e^{1.25}$$

$$\approx \$69806.86$$

110

(110) The half-life for a Facebook post is 90 minutes (1.5 hrs). Write an exponential function F that gives the percentage of engagements remaining on a typical Facebook link after t hours. Estimate this percentage after 5 hours.

$$F(t) = 100\left(\frac{1}{2}\right)^{t/1.5} \text{ percent}$$

because the half-life is 1.5 hours

$$F(5) = 100\left(\frac{1}{2}\right)^{5/1.5} \approx 9.92\%$$

Section 5.4

Section 5.4

In Exercises 111-112, convert the given equation as indicated.

(111) $\log_a(x-1) = 3$ (to exponential form)

111

$a^3 = x - 1$ (exponential form)

112

(112) $a^{10x} = 2$ (to logarithmic form)

$\log_a 2 = 10x$ (logarithmic form)

In Exercises 113-115, solve each logarithmic equation algebraically. Round to 4 decimal places when appropriate.

(113) $\log_3 x = 4$ (114) $\log 4x = 1$ (115) $\ln x = -3$

 # 113

$3^4 = x$ (exponential form)

$x = 3^4$

$x = 81$

 # 114

$10^1 = 4x$ (exponential form)

$4x = 10$

$x = 2.5$

 # 115

$e^{-3} = x$ (exponential form)

$x = e^{-3}$

$x \approx 0.0498$

In Exercises 116-118, solve each exponential equation algebraically. Round to 4 decimal places when appropriate.

(116) $2^x = 7$ (117) $10^{4x} = 3$ (118) $e^x = 0.9$

 # 116

$\log_2 7 = x$ (logarithmic form)

$x = \log_2 7$

$x = \frac{\log 7}{\log 2}$ (Change of base formula)

$x \approx 2.8074$

 # 117

$\log_{10} 3 = 4x$ (logarithmic form)

$4x = \log 3$

Divide both sides by 4

$x = 0.25 \log 3$

$x \approx 0.1193$

 # 118

$x = \ln 0.9$ (logarithmic form)

$x \approx \ln 0.9$

 # 119

(119) For $f(x) = \ln(x+1) - 2$, find its vertical asymptote, domain, two "identity" points and all intercepts. Then sketch the graph.

The expression inside the parenthesis cannot be negative

$x + 1 > 0$

$x > -1$

The domain is $(-1, \infty)$

Based on the domain, the vertical asymptote is $x = -1$

$f(0) = (\ln 1) - 2 = 0 - 2 = -2$

The y-intercept is $(0, -2)$

Solve for x in

$f(x) = 0$

$\ln(x + 1) - 2 = 0$

$\ln(x + 1) = 2$

$e^2 = x + 1$

$x = e^2 - 1$

The x-intercept is $(e^2 - 1, 0)$ where $e^2 - 1 \approx 6.3891$

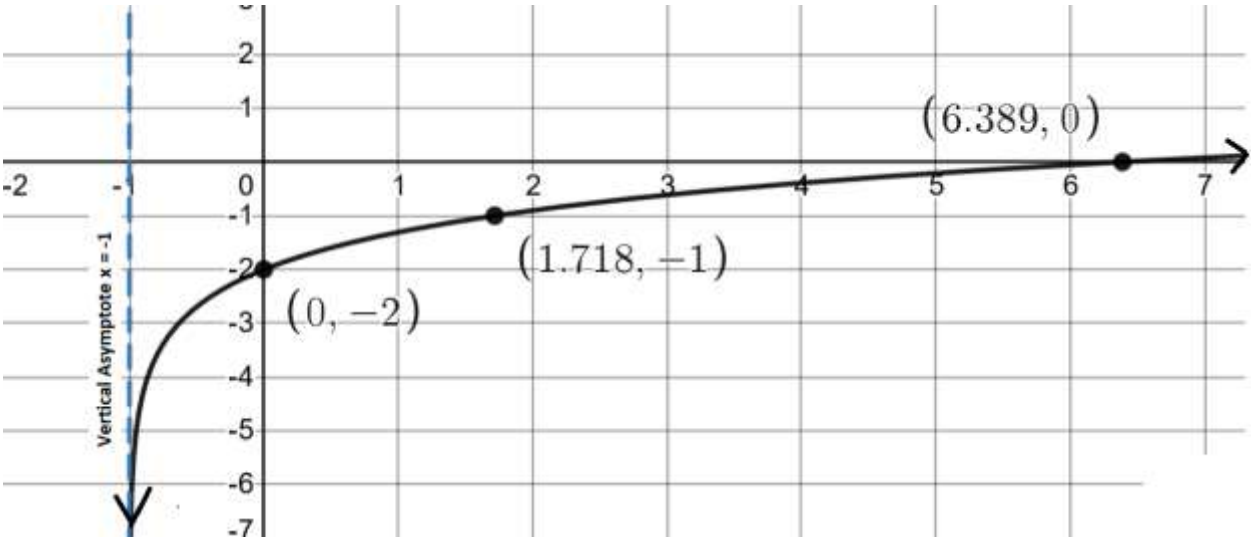
Solve for x:

x+1 = 1 and x+1 = e

x = 0 and x = e-1

(0, -2) and (e-1, -1) are the identity points where $e - 1 \approx 1.718$

Graph:



 # 120

(120) The barometric air pressure in inches of mercury at a distance of x miles from the eye of a severe hurricane is modeled by $f(x) = 0.48 \ln(x+1) + 27$.

(a) Evaluate $f(10)$ and interpret your result.

(b) At what distance from the eye is the air pressure 27.5 inches of mercury?

Part-a

$f(10) = 0.48 \ln(11) + 27 \approx$ **28.15 inches of mercury**

We predict using the model that the barometric air pressure at a distance of 10 miles from the eye of a severe hurricane is about 28.15 inches.

Part-b

Solve for x

$f(x) = 27.5$

$0.48 \ln(x + 1) + 27 = 27.5$

Subtract 27 from each side

$0.48 \ln(x + 1) = 0.5$

Divide both sides by 0.48

$$\ln(x+1) = \frac{0.5}{0.48}$$

$e^{0.5/0.48} = x + 1$ (exponential form)

Subtract 1 from each side

$e^{0.5/0.48} - 1 = x$

$x = e^{0.5/0.48} - 1$

$x \approx$ **1.8339 miles**